

16.5 Solving Quadratics in the Form $ax^2 + bx + c = 0$ when $a \neq 1$

Solve each equation by completing the square.

1. $2x^2 - 4x - 7 = 5$

$$2(x^2 - 2x + 1) = 12 + 2$$

$$2(x-1)^2 = 14$$

$$(x-1)^2 = 7$$

$$x-1 = \pm\sqrt{7}$$

$$x = 1 \pm \sqrt{7}$$

$$x = 1 + \sqrt{7}, 1 - \sqrt{7}$$

3. $3x^2 + 6x - 15 = 0$

$$3(x^2 + 2x + 1) = 15 + 3$$

$$3(x+1)^2 = 18$$

$$(x+1)^2 = 6$$

$$x+1 = \pm\sqrt{6}$$

$$x = -1 \pm \sqrt{6}$$

$$x = -1 + \sqrt{6}, -1 - \sqrt{6}$$

5. $5x^2 - 10x - 8 = 27$

$$5(x^2 - 2x + 1) = 35 + 5$$

$$5(x-1)^2 = 40$$

$$(x-1)^2 = 8$$

$$x-1 = \pm\sqrt{8}$$

$$x = 1 \pm 2\sqrt{2}$$

$$x = 1 + 2\sqrt{2}, 1 - 2\sqrt{2}$$

7. $2x^2 - 16x = 14$

$$2(x^2 - 8x + 16) = 14 + 32$$

$$2(x-4)^2 = 46$$

$$(x-4)^2 = 23$$

$$x-4 = \pm\sqrt{23}$$

$$x = 4 \pm \sqrt{23}$$

$$x = 4 + \sqrt{23}, 4 - \sqrt{23}$$

9. $4x^2 + 12x + 5 = 0$

$$4(x^2 + 3x + \frac{9}{4}) = -5 + 9$$

$$4(x + \frac{3}{2})^2 = 4$$

$$(x + \frac{3}{2})^2 = 1$$

$$x + \frac{3}{2} = \pm 1$$

$$x = -\frac{3}{2} \pm 1$$

$$x = -\frac{1}{2}, -\frac{5}{2}$$

2. $3x^2 - 18x - 3 = 21$

$$3(x^2 - 6x + 9) = 24 + 27$$

$$3(x-3)^2 = 51$$

$$(x-3)^2 = 17$$

$$x-3 = \pm\sqrt{17}$$

$$x = 3 \pm \sqrt{17}$$

$$x = 3 + \sqrt{17}, 3 - \sqrt{17}$$

4. $-2x^2 + 8x + 18 = 0$

$$-2(x^2 - 4x + 4) = -18 - 8$$

$$-2(x-2)^2 = -26$$

$$(x-2)^2 = 13$$

$$x-2 = \pm\sqrt{13}$$

$$x = 2 \pm \sqrt{13}$$

$$x = 2 + \sqrt{13}, 2 - \sqrt{13}$$

6. $3x^2 + 12x = 18$

$$3(x^2 + 4x + 4) = 18 + 12$$

$$3(x+2)^2 = 30$$

$$(x+2)^2 = 10$$

$$x+2 = \pm\sqrt{10}$$

$$x = -2 \pm \sqrt{10}$$

$$x = -2 + \sqrt{10}, -2 - \sqrt{10}$$

8. $4x^2 - 20x - 3 = 0$

$$4(x^2 - 5x + \frac{25}{4}) = 3 + 25$$

$$4(x - \frac{5}{2})^2 = 28$$

$$(x - \frac{5}{2})^2 = 7$$

$$x - \frac{5}{2} = \pm\sqrt{7}$$

$$x = \frac{5}{2} \pm \sqrt{7}$$

$$x = \frac{5}{2} + \sqrt{7}, \frac{5}{2} - \sqrt{7}$$

10. $9x^2 + 36x = -11$

$$9(x^2 + 4x + 4) = -11 + 36$$

$$9(x+2)^2 = 25$$

$$(x+2)^2 = \frac{25}{9}$$

$$x+2 = \pm\frac{5}{3}$$

$$x = -2 \pm \frac{5}{3}$$

$$x = -\frac{1}{3}, -\frac{11}{3}$$