Name:______________________________________ Date:_________
Pre-Calculus Pre-AP 4.5 Sinusoidal Problems Worksheet
Do on your own paper.

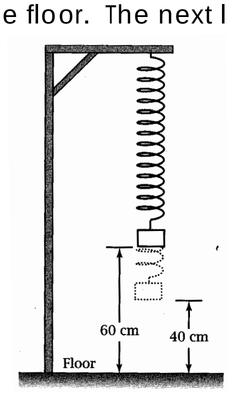
1. Mark Twain sat on the deck of a river steamboat. As the paddle wheel turned, a point on the blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When Twain's stopwatch read 4 seconds, the point was at its highest, 16 ft above the water's surface. The wheel's diameter was 18 ft, and it completed a revolution every 10 seconds.

a. Sketch a graph of the sinusoid.
b. Write the equation of the sinusoid.
c. How far above the surface was the point when Mark's stopwatch read 17 seconds?
d. What is the first positive value of "t" at which the point was at the water's surface? At that time, was the point going into or coming out of the water? How can you tell?

2. Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time t = 0 years. A minimum number of 200 foxes appeared when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

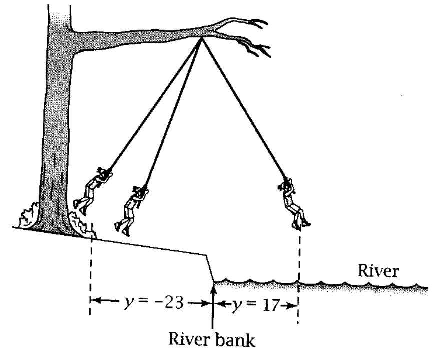
a. Sketch a graph of this sinusoid.
b. Write an equation expressing the number of foxes as a function of time.
c. Predict the fox population when t = 7, 8, 9, and 10 years.
d. Foxes are declared an endangered species when their population drops below 300. Between what two nonnegative values of t were the foxes first endangered?

3. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. Start a stopwatch. When the stopwatch reads 0.3 sec, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 sec.



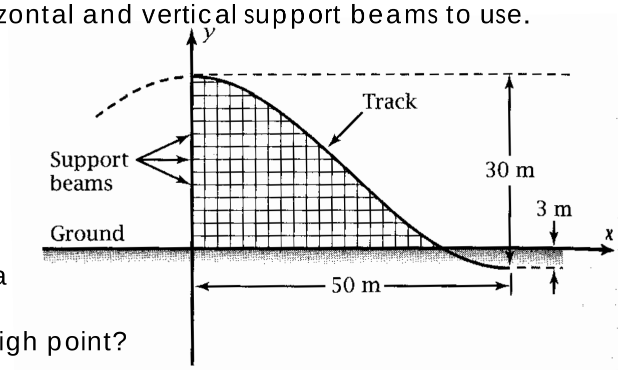
a. Sketch the graph of this sinusoidal function.
b. Write an equation expressing the distance from the floor as a function of time.
c. What is the distance from the floor when the stopwatch reads 17.2 sec?
d. What was the distance from the floor when you started the stopwatch?
e, What is the first positive value of time when the weight is 59 cm above the floor?

4. Zoey is at summer camp. One day she is swinging on a rope tied to a tree branch, going back and forth alternately over land and water. Nathan starts a stopwatch. When x = 2 seconds, Zoey is at one end of her swing, y = −23 feet from the river bank. When x = 5 seconds, she is then at the other end of her swing, y = 17 feet from the riverbank. Assume that while she is swinging y varies sinusoidally with x.



a. Sketch the graph of y versus x and write an equation.
b. Find y when x = 13.2 sec. Was Zoey over land or over water at this time?
c. Find the first positive time when Zoey was directly over the river bank (y = 0).

5. A theme park is building a portion of a roller coaster track in the shape of a sinusoid. You have been hired to calculate the lengths of the horizontal and vertical support beams to use.



a. The high and low points of the track are separated by 50 m horizontally and 30 m vertically. The low point is 3 m below the ground. Let y be the distance (in meters) a point on the track is above the ground. Let x be the horizontal distance (in meters) a point on the track is from the high point. Find a particular equation for y as a function of x.
b. How long is the vertical support beam at the high point? At x = 4 m? At x = 32 m?
c. How long is the horizontal support beam that is 25 m above the ground? At 5 m above the ground?
d. Where does the track first go below ground?

6. Suppose you seek a treasure that is buried in the side of a mountain. The mountain range has a sinusoidal vertical cross section. The valley to the left is filled with water to a depth of 50 meters, and the top of the mountain is 150 meters above the water level. You set up an x-axis at the water level and a y-axis at 200 meters to the right of the deepest part of the water. The top of the mountain is at x = 400 meters.

a. Write the equation expressing y for points on the surface of the mountain as a function of x.
b. Show algebraically that the sinusoid in part a contains the origin (0, 0).
c. The treasure is located beneath the surface at the point (130, 40). Which would be a shorter way to dig to the treasure, a horizontal tunnel or a vertical tunnel? Show your work.