Surface Area
Think Green

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/
Retell, Use Manipulatives, Activating Prior Knowledge, Self/
Peer Revision

The Horticulture Club has been given some land to build a
greenhouse. The club president, Lillie, has to design a greenhouse
and determine how much building material is needed. She will
investigate possible designs by building models. Her teacher has
provided several nets for the models Lillie wants to explore.
Lillie decides to begin her investigation with prisms.

> **CONNECT TO SCIENCE**
>
> *Horticulture* is the science and
> art of growing fruit, flowers,
> ornamental plants, and
> vegetables.

1. Cut out Net 1, Net 2, and Net 3 on pages 273 and 275. Build the
   models. Write the best name for each prism.
   **Net 1: triangular prism; Net 2: pentagonal prism; Net 3:
   rectangular prism**

2. Compare and contrast the characteristics of the prisms built
   from the nets provided.
   **Answers may vary. Sample answer: They all have two bases,
   their faces are rectangles, but the shapes of the bases are
   different.**

3. Explain how to calculate the surface area of a prism.
   **Answers may vary. Sample answer: Find the sum of the areas of
   all of the faces and bases.**

> **MATH TIP**
>
> Measure the Net 1 prism by using
> the square units on its surface.
>
> Measure the Net 2 prism by
> using the measures given on its
> surface.
>
> Measure the Net 3 prism with an
> inch ruler.

4. Measure the nets to determine the surface area of each prism.
   Explain how you found the surface area.
   **Explanations may vary. Sample answer: Triangular prism: 544
   square units; pentagonal prism: 1320 square inches; rectangular
   prism: 15.125 square inches; I found the area of each face and
   then added the six areas.**

While doing online research about greenhouse designs, Lillie found
the formula $SA = Ph + 2B$ for determining surface area of a prism
where P represents the perimeter of the base, B represents the area
of the base, and h represents the height of the prism.

5. Verify that the formula Lillie found will correctly calculate the
   surface area of the prisms you made in Item 1. Record the mea-
   sures in this table and use the formula to find the surface areas.
   Then compare them to those you found in Item 4.

Net	P	h	B	SA
1	32 units	14 units	48 units^2	544 units^2
2	50 units	20 units	160 units^2	1320 units^2
3	8.5 units	0.75 units	4.375 units^2	15.125 units^2

for the class to avoid confusion.

5 **Self Revision/Peer Revision** For each prism built in
Question 1, students list the measurements necessary to use
the formula and then show their calculations.
For example, for the triangular prism:

$$P = 22 \text{ units}, h = 14 \text{ units}, B = 48 \text{ units}^2$$
$$SA = (22 \text{ units})(14 \text{ units}) + 2(48 \text{ units}^2) = 404 \text{ units}^2$$

ACTIVITY 4.4 Investigative

Surface Area

Activity Focus
- Investigating relationships
  between the area of two-
  dimensional nets and the
  surface area of three-dimensional
  figures
- Applying formulas for determining
  the surface area of solids

Materials
- Rulers
- Scissors
- Calculators
- Grid paper
- BLM 4, 5, 6 (optional)
- Cylindrical cans

Chunking the Activity

#1–2	#9	#21
#3	#10	#22
#4	#11–12	#23–24
#5	#13	#25–26
#6	#14–16	#27–28
#7	#17–18	#29
#8	#19–20	#30

1 **Use Manipulatives,
Summarize/Paraphrase/Retell**
The purpose of Items 1–4 is
to activate prior knowledge
of prisms. Students make
connections between the area of
a net and the surface area of a
solid built from the net.

2 **Activating Prior Knowledge**
Let students work independently
to make a list before comparing
answers with a partner and the
group.

3 **Activating Prior Knowledge**
Students should recall from the
activity "All Boxed Up" how to
determine surface area by finding
the sum of the areas of the sides.

4 **Use Manipulatives** Students
will use different methods to
determine the surface area of each
net. They use the grid on the
triangular prism in Figure 1, the
measurements given in Figure 2,
and measure Figure 3, using a ruler.
Be sure to specify units of measure

6 Think/Pair/Share The purpose here is to have students read and use measurements correctly when given a three-dimensional drawing of a prism rather than a two-dimensional net. Again, it will be helpful for students to list the measurements to use before applying the surface area formula.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 272, #1–3

UNIT 4 PRACTICE
p. 290, #21–22

7 Activating Prior Knowledge, Think/Pair/Share The purpose of Items 7–10 is for students to make connections between the area of the net for a pyramid and the surface area of the pyramid. Without building the pyramids, students should recognize the faces of the pyramids are triangles and that there is only one base. The number of faces is determined by the number of sides of the base.

Add the term *slant height* to the Interactive Word Wall.

8 Use Manipulatives

9 Interactive Word Wall, Use Manipulatives In this question, students should recognize that the slant height of the pyramid is the height of the triangular faces. It is longer than the height of the pyramid. The height of the pyramid is the shortest distance from the vertex of the pyramid to the middle of the base.
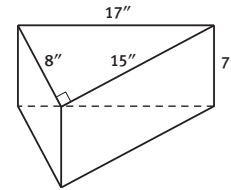
10 Use Manipulatives Students should use the given grid to determine the measure of the sides on the pyramid.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Activating Prior Knowledge, Use Manipulatives, Interactive Word Wall

6. Use the formula Lillie found ($SA = Ph + 2B$) to determine the surface area of this triangular prism.

$P = 40$, $h = 7$, $B = 60$, $SA = (40)(7) + 2(60) = 400$ **square inches**

On page 277 are nets of pyramids that Lillie's teacher gave her.

7. Compare and contrast the characteristics of the pyramids that could be built from Net 4 and Net 5.

Answers may vary. Sample answer: Both nets have one base and their faces are triangles, but the shapes of the bases are different.

8. Cut out Net 5 and fold it to make a model of a square pyramid.

The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base. The height of a triangular face is the **slant height** of the pyramid.

9. Consider the slant height of the square pyramid.

  a. Draw the slant height on the faces of the pyramid you made.
  The slant height for each of the four faces is a line segment from the top vertex of the triangle to the center of the base.

  b. Why do you think it is called the slant height?
  Answers may vary. Sample answer: It is slanted instead of being vertical.

  c. How could you use your model to show the height of the pyramid?
  Answers may vary. Sample answer: I could put a pencil on the center of the base and hold it so that the four vertices touch it.

10. Use the net to calculate the surface area of the square pyramid you built. Show the calculations that lead to your answers.

340 square units;
$4 \cdot \left(\frac{1}{2}\right)(10 \cdot 12) + (10 \cdot 10) = 4 \cdot \left(\frac{1}{2}\right)(10 \cdot 12) + (10 \cdot 10)$
$= (4 \cdot 60) + 100 = 240 + 100 = 340$ **square units**

SUGGESTED LEARNING STRATEGIES: Create Representations, Debriefing, Self/Peer Revision, Use Manipulatives

11. Determine the number of triangular faces on a pyramid whose base has *n* sides.
    n

12. A formula for finding the surface area of a **regular pyramid** uses four variables:

    B to represent the area of the base,

    l to represent slant height,

    s to represent length of a base edge and

    n to represent the number of sides on the base.

    Write an equation, in terms of these variables, for finding the surface area of a regular pyramid.
    Surface Area $= B + \frac{1}{2}nsl$

13. Use the equation you wrote in Item 12 to calculate the surface area of the model you made of the square pyramid. Confirm that your answer agrees with the answer you got in Question 10.
    $B = 100, n = 4, s = 10, l = 12, SA = 100 + \frac{1}{2}(4)(10)(12) = 340$

In her research, Lillie discovers that The Myriad Botanical Gardens in Oklahoma City has a greenhouse in the shape of a cylinder. She wants to consider a cylindrical shape for their greenhouse. Help Lillie investigate the surface area of a cylinder.

14. Trace the top and bottom of a can on grid paper. Cut out the shapes drawn.

15. What shape is a piece of paper that would cover the side of the can?
    rectangle

16. Describe the relationship between the circumference of the base of the can and the length of the paper that covers the side of the can.
    They are the same.

17. From the grid paper cut out a piece of paper that would cover the side of the can.

18. Use the shapes you have cut out to make a net that could be used to make a cylinder.
   **The net should look something like the illustration to the right.**

My Notes

> **MATH TERMS**
> A **regular polygon** is a polygon whose sides are all congruent and whose angles are all congruent.
>
> A **pyramid** is a three-dimensional figure whose base is a polygon and whose other faces are triangles that share a common vertex.
>
> A **regular pyramid** has a base that is a regular polygon.

ACTIVITY 4.4 *Continued*
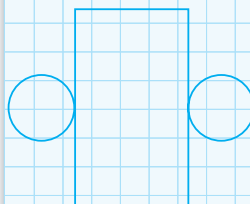
11-12 Create Representations, Debriefing It may be helpful to have models of several different types of pyramids (hexagonal, pentagonal, octagonal) available to use for discussion of these questions.

13 Self/Peer Revision Students should make a list of the measurements needed to apply the formula before verifying their answer. Encourage students to confirm their results by comparing their answer with the answer they got in Item 10 and making revisions when necessary.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 272, #4

UNIT 4 PRACTICE
p. 290, #23–24

14 Use Manipulatives Each student should have a cylindrical can for this part of the activity. The purpose of Items 14–19 is for students to investigate the relationships between the measures of the bases of a cylinder and the measures of the side. Students will build a net with pieces they have created using a cylindrical can and grid paper.

15 It may be helpful to illustrate the shape of the piece of paper that would cover the side of the can by carefully peeling the label off a can. Explain that the measurements of this rectangle are not exactly the surface area since the label allows for overlapping the edges.

19 Use Manipulatives For checking purposes, students should record the radius and height of each can.

20-21 Debriefing, Create Representations Students will develop a rule for determining the surface area of a cylinder knowing the radius of the base and the height of the cylinder.

Differentiating Instruction

To help students visualize the surface area of a cylinder, have them to label the length of the rectangle used to cover the side of the cylinder with $2\pi r$ (circumference of the circular base) and to label its width h.

22 Identify a Subtask, Think/ Pair/Share Students should identify the radius and height of each cylinder. Since the method for determining the solution is just as important as a correct answer, remind students to organize their work in a way that that clearly shows which cylinder belongs with which calculations.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 272, #5

UNIT 4 PRACTICE
p. 290, #25

23-24 Use Manipulatives The purpose of Items 23–28 is to develop a method for determining the surface area of a cone. In building the cone from the net students should recognize that the circumference of the smaller circle is the arc length of the portion of the larger circle that remains.

My Notes

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Debriefing, Create Representations, Think/Pair/Share, Identify a Subtask

19. Use your net to estimate the surface area of the cylindrical can.
Answers may vary based on the size of the can.

20. Explain how to determine the surface area of a cylinder.
Answers may vary. Sample answer: The surface area will be the sum of the areas of both circles and the area of the rectangle.

21. Use the variables r and h to represent the radius and height of a cylinder. Write an equation, in terms of r and h, for determining the surface area of a cylinder.
$SA = 2\pi r^2 + 2\pi rh$

22. The greenhouse at the Myriad Botanical Gardens in Oklahoma City is a large cylinder with a diameter of 70 feet and a height of 224 feet. Lillie thinks that the club might build a smaller version that would have a diameter of 10 feet and a height of 20 feet. Determine the surface area of these two cylinders.

Calculations may vary. Sample answer:
Surface area of large cylinder
$$= \left(2 \cdot \frac{22}{7} \cdot 35^2\right) + \left(2 \cdot \frac{22}{7} \cdot 35 \cdot 224\right)$$
$$= (2 \cdot 22 \cdot 5 \cdot 35) + (2 \cdot 22 \cdot 5 \cdot 224) = 56{,}980 \text{ ft}^2;$$
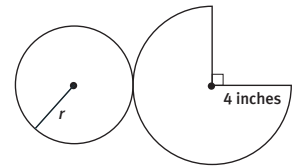**Surface area of small cylinder**
$$= (2 \cdot 3.14 \cdot 5^2) + (2 \cdot 3.14 \cdot 5 \cdot 20)$$
$$= (50 \cdot 3.14) + (200 \cdot 3.14)$$
$$250 \cdot 3.14 = 785 \text{ ft}^2$$

After considering prisms, pyramids, and cylinders, Lillie wants to study one last solid before designing the greenhouse. One of the nets her teacher gave her was for a cone. It looks like this:

MATH TERMS

A **sector** is a part of a circle that is formed by 2 radii and part of the circumference of the original circle.

23. Cut out Net 6 on page 279, and build the cone.

24. The larger figure in the net is a **sector**. It looks like a circle that is missing a part. Why is the sector part of the net?
Answers may vary. Sample answer: The shape will form a cone when the two 4-inch lengths are connected together.

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Quickwrite

25. How much of the circle remains in the sector? Explain your answer.
Explanations may vary. Sample answer: The larger figure is $\frac{3}{4}$ of a circle; a circle has 360° and 90° have been cut out; 90° is $\frac{1}{4}$ of a circle, so $1 - \frac{1}{4} = \frac{3}{4}$.

26. How is the circumference of the smaller circle related to the curved edge of the sector?
The circumference of the smaller figure is the same as the length of the curved edge of the larger figure.

27. Calculate the circumference of the smaller circle. Give the answer in terms of π. Then find the radius of the smaller circle.
circumference = 6π, radius = 3

28. Determine the surface area of the cone. Show your work. Give the answer in terms of π.
Area of sector: $\frac{3}{4} \cdot \pi \cdot 4^2 = 12\pi$; Area of base: $\pi \cdot 3^2 = 9\pi$
Total area of cone: $12\pi + 9\pi = 21\pi$

Lillie knows that she will have to use three railroad ties to build steps up to the piece of land on which the greenhouse will be built. The ties are assembled as shown to the right and form a complex solid.

29. If each railroad tie measures 1 ft by 1 ft by 8 ft, what is the surface area of the assembled steps?
Answers may vary. If students consider all faces, the surface area is 70 sq. ft; if they consider only the parts exposed to light, they may answer 38 square ft.

30. Write a letter to Lillie giving her your suggestion for the design of a greenhouse. In your letter tell her what solids you would use and why you selected them. Include a drawing or sketch showing the dimensions of the greenhouse along with calculations for surface area.
Answers may vary. Sample answer: Dear Lillie: Now you can look at the ways that the surface areas of solids relate to their volumes. You must decide whether you want more surface area so there will be more light for the plants or less surface area so that the building costs will be less. I recommend a triangular prism that is 8 feet in height, 12 feet wide, and 30 feet long placed on a rectangular prism that is 12 feet wide, 4 feet high, and 30 feet long. The prism sides will be 10 feet by 30 feet, so the surface area of the two slanted sides will be 600 square feet, and the surface area of the front and back triangles will be 96 square feet. The exposed surface area of the rectangular prism will be 336 square feet, for a total surface area of 1032 square feet. See my sketch below. Sincerely, A Friend
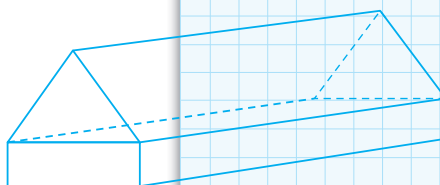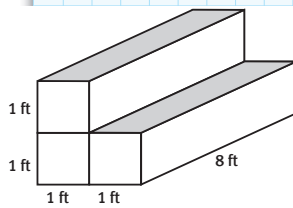
27 Identify a Subtask This is a two-part question. Students must first determine the circumference of the smaller circle by finding the length of the remaining part of the circumference of the larger circle.
$C = \frac{3}{4}(2\pi(4)) = 6\pi$
Since the circumference of the smaller circle is 6π, the radius of the circle must be 3.

28 Quickwrite Students should recognize that the surface area of the cone is the sum of the area of the smaller circle and the portion of the larger circle that remains after the piece is taken out.
$9\pi + \frac{3}{4}(16\pi) = 21\pi$

29 Encourage students who give different answers based on their understanding of the situation to discuss their reasoning.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 272, #6–8

UNIT 4 PRACTICE
p. 290, #26

CHECK YOUR UNDERSTANDING

1. Answers may vary. Sample answer: 2' × 2' × 11'

2. 240 mm²

3. 152 square feet

4. 54.25 ft²

5. $560\pi \approx 1758.4$ in.²

6. 36π cm²

7. 104 in.²

8. Answers may vary. Possible answers: when painting an object or constructing the outer walls of a house.


My Notes

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. The surface area of a rectangular prism is 96 square feet. Give one possible set of dimensions for this prism. You can use either of two formulas to find the surface area of a rectangular prism:
$SA = 2lw + 2lh + 2hw$ or $SA = Ph + 2B$.

2. Determine the surface area of the trapezoidal prism.
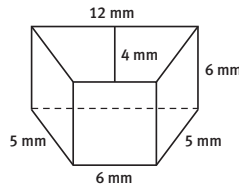
3. A tent with canvas sides and floor is shown below. How much canvas is in the sides and floor of the tent?
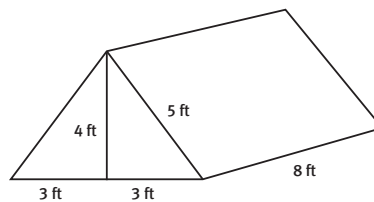

4. Determine the surface area of the pyramid.
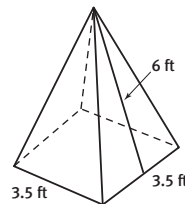
5. Find the surface area of the cylinder to the nearest tenth of an inch. Use 3.14 to approximate π.
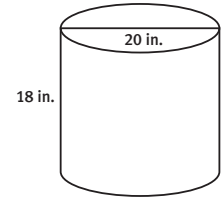
6. Find the surface area of the cone that can be formed from the net below. The sector is $\frac{4}{5}$ of a circle.
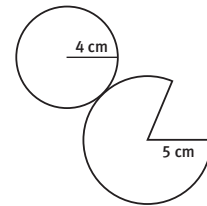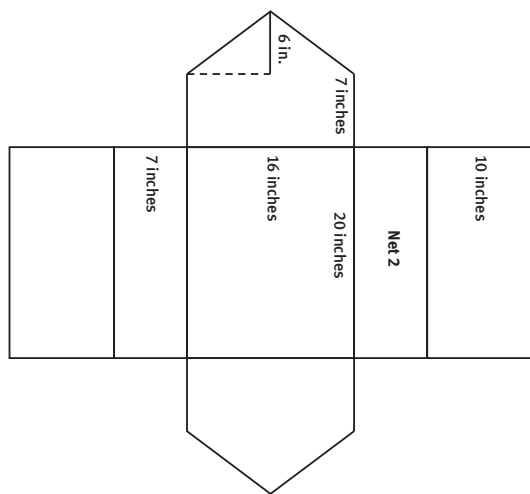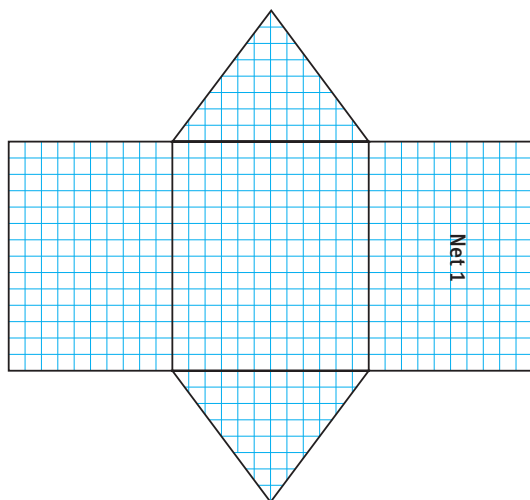

7. This composite figure consists of a regular pyramid and a cube. The edge of the cube is 4 inches. The height of each triangular face of the pyramid is 3 inches. What is the surface area of the figure?
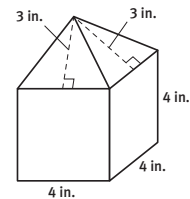
8. **MATHEMATICAL REFLECTION** What are some real-world situations for which it would be important to know the surface area of an object.
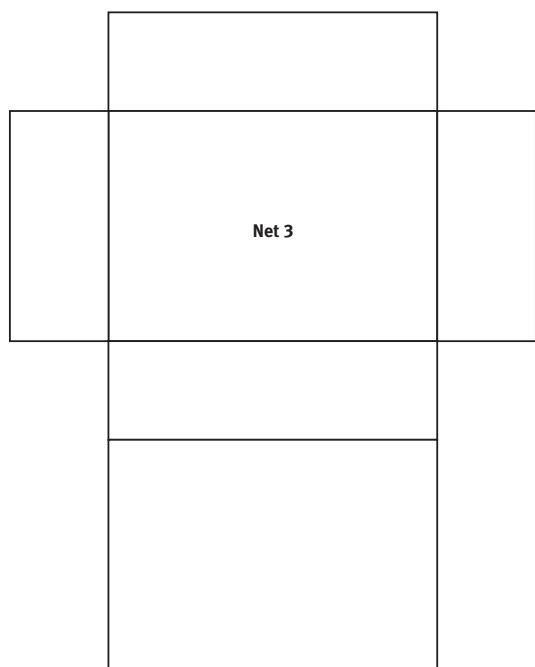
ACTIVITY 4.4 *Continued*

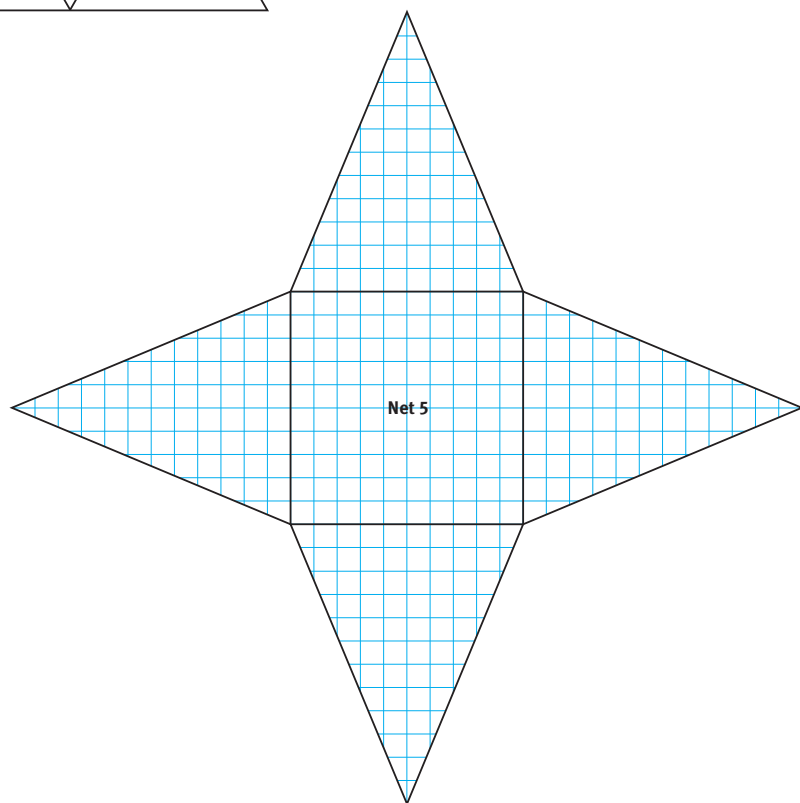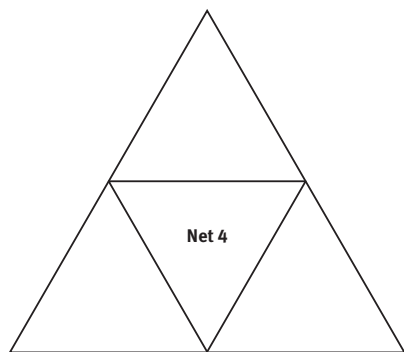Students will cut out and use the two nets on this student page during the activity.

Net 1

6 in.

7 inches

7 inches

16 inches

20 inches

Net 2

10 inches

Net 3

ACTIVITY 4.4 *Continued*

Students will cut out and use the net on this student page during the activity.

Net 4

Net 5

ACTIVITY 4.4 *Continued*

Students will cut out and use the two nets on this student page during the activity.

Net 6

ACTIVITY 4.4 *Continued*

Students will cut out and use the net on this student page during the activity.