I Can Name that Angle in One Measure! – Grade Eight

Ohio Standards Connection:

Geometry and Spatial Sense

Benchmark C
Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.

Indicator 2
Recognize the angles formed and the relationship between the angles when two lines intersect and when parallel lines cut by a transversal.

Benchmark H
Establish the validity of conjectures about geometric objects, their properties and relationships by counter-examples inductive and deductive reasoning, and critiquing arguments made by others.

Indicator 1
Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.

Measurement

Related Benchmark E
Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.

***Lesson Summary**:*
Students make conjectures and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines cut by a transversal. A pre-assessment assesses and reinforces students' use of terminology related to characteristics of angles and lines. Students then develop skills in identifying and using angle relationships to find the measures of angles.

***Estimated Duration**: Three hours to three hours and 30 minutes*

Commentary:
Understanding angle measure is a critical foundational skill for this lesson. Students need to understand that the measure of an angle is a description of the region between the rays, rather than the lengths of the rays. Sometimes this notion is not clear to all students. For example, students learn to read a protractor with little attention to the scale or measure which quantifies a turn around a point (the vertex). Consider asking students to create an angle using a pipe cleaner or drink stirrers with a twist tie hinge and to demonstrate how to find the measure using a clear protractor and overhead projector. Ask students to first decide if an angle is acute or obtuse visually and then read the scale that "matches" the classification.

Pre-Assessment:
The pre-assessment assesses students' familiarity with terminology related to angles and lines (e.g., acute, obtuse, right, intersecting, parallel, perpendicular, oblique, vertical angles, straight angles, supplementary and complementary angles.)

- Provide students with an activity sheet that includes illustrations of three sets of lines: a pair of intersecting lines, a pair of perpendicular lines and a pair of parallel lines cut by a transversal. A transparency of the activity sheet may be used in place of or in addition to individual activity sheets. See Attachment A, *Pre-Assessment Activity*.

I Can Name that Angle in One Measure! – Grade Eight

Related Indicator 8
Find the sum of the interior and exterior angles of regular convex polygons with and without measuring angles with a protractor.

Mathematical Processes Benchmark
D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

- Ask students to write as many true statements as possible about the lines and angles. Students may use protractors during this activity or other strategies to compare and verify the measures of angles; e.g., cut paper to "fit" an angle and use it to find angles of similar measure.
- Organize students into groups of three or four to share and compare their statements. Have each group write its statements on chart paper.
- Provide idea starters to broaden the ways students think about the angles and lines through probing questions; e.g., What can be said about angles 1 and 2? Do any angles appear to be congruent?
- Have groups share one statement from their lists. Record the statement on a transparency or by "checking" the statement on the lists/chart paper. Continue asking groups for "new" statements in a "round robin" manner. Students may "challenge" a statement by asking for verification of its accuracy or when it restates one previously shared. The group making the statement must then provide justification for the statement shared.
- Inform students that they can only share statements that have not been shared by another group. Continue asking each group to share statements from their lists until all statements have been shared.

Scoring Guidelines:
Determine the level of familiarity with and use of terminology related to angles and lines. The process provides valuable information for the teacher to use in identifying intervention needs and modifying the lesson to address students' prior knowledge and skills. Individual student scores are not required; however, a sample scoring guideline is provided in Attachment B, *Sample Scoring Guidelines for Pre-assessment*.

Post-Assessment:
The post-assessment includes three types of items/tasks:
- A set of statements about angle relationships in illustration to which students agree or disagree and provide justification for their responses.

- Illustrations of a situation and the measure of one angle for students to find the measures of designated angles.
- A real-world application problem involving angle and line relationships.

Students complete the post-assessment independently. See Attachment C, *Post-Assessment.*

Scoring Guidelines:
Customize scoring guidelines for each type of item.

- One point may be awarded for each correct response for the task for which students are asked to find the measures of angles using an illustration of a situation and the measure of one angle.
- A two-point scoring rubric may be used to assess performance on each of the statements about angle relationships for which students classify as always true, sometimes true or never true and provide justification for their response. The scoring guidelines focus on the accuracy of the classification and the quality of the supporting explanation or work.
- A three-point scoring rubric may be used to assess student performance on the application problem.

See Attachment D, *Post-Assessment Answer Key and Sample Scoring Guidelines.*

Instructional Procedures:

Part One:

1. Complete Pre-Assessment.
2. Generate a list of 6-10 conjectures to investigate. Draw conjectures from the statements made during the pre-assessment and supplement these to ensure all the basic angle relationships are reflected and that some conjectures are invalid. Students indicate whether each conjecture is always, sometimes or never true and provide supporting evidence for their decision, such as a drawing of the situation or a counter-example. See Attachment E, *Conjecture Form*, for sample activity sheet that can be used by students to record information.
3. Provide students with electrical wire ties, uncooked spaghetti, pipe cleaners or straws to use as models for lines. (Students may need tape to secure straws so they do not roll.)
4. Ask students to use the materials to model and verify the conjecture and record their responses.
5. Organize students into groups of two to four to discuss their conjectures. Groups must reach consensus about the validity of each conjecture and provide supporting evidence for their decisions.
6. Lead the class in debriefing about the results of their investigations and identify conjectures for which no class consensus about the validity of the conjecture was achieved.
7. Provide the opportunity for groups with opposing views to present their reasoning and supporting evidence.

8. Close this part of the lesson by recording the conjectures agreed upon by consensus on chart paper or by having students write in their mathematics journal what they know about angle relationships in situations involving intersecting, parallel and perpendicular lines.

Part Two:

9. Write questions related to angle relations on sentence strips and post them in the front of the room. Examples of questions that can be used during this activity include:
   - Are corresponding angles always congruent?
   - Are vertical angles always congruent?
   - Are adjacent angles formed by intersecting lines always supplementary?
   - When parallel lines are cut by a transversal, what special angle relationships are created and do they also hold true if the lines are not parallel and when lines are perpendicular?
   - Is it possible to create intersecting lines with adjacent angles that are not supplementary?
   - Can you find these angle relationships in games or in our classroom?
   - Do you have to have a protractor to find an angle's measure?
   - What are the advantages of knowing two lines are parallel?
10. Ask students to individually think about and answer each question.
11. Facilitate a reporting out of answers. Encourage students to evaluate each others' answers and comments. Alternatively, ask students to respond to a subset of the questions as a journal writing and/or homework assignment.
12. Introduce using symbols to represent the relationships; e.g., translate worded descriptions to use of symbols, such as two angles are supplementary to $m\angle 1 + m\angle 2 = 180$.
13. Create a graphic organizer to summarize the relationships among angles in intersecting, perpendicular and parallel lines cut by a transversal. Introduce new terminology related to angles, such as corresponding, alternate interior, alternate exterior, same side interior, same side exterior, vertical, as students describe the relationships. A table or web could be used as a graphic organizer. See Attachment F, *Graphic Organizer,* for an example of a table format that can be used to organize information about the relationships.
14. Have students identify examples of angle relationships found in the classroom.
15. Ask students to identify two real-world situations that illustrate the concept in each cell of the graphic organizer. Students may bring in pictures from newspapers or magazines, sketches or descriptions; e.g., the railing and balusters on the school's stairs.
16. Facilitate a whole class debriefing and sharing of the real-world examples identified and ways to represent those situations using sketches and mathematics symbols by students. Record examples by category on chart paper or the board. Invite students to add additional real-world examples as the lesson progresses.

Part Three:

17. Present the students with a real-world situation where one has to find the measure of an angle that cannot be measured directly using a protractor, such as cutting drywall to fit around angled wall, fireplace or vaulted ceiling. The problem should include restrictions that require students to use angle relationships to solve the problem. See Attachment G, *Staircase Activity*, for a scenario that can be used or select a similar scenario, such as cutting tile to fit around a cabinet or other situations can be found in the school, home or local setting.

18. Have students brainstorm ideas for solving the problem for three to five minutes.
19. Ask students to solve the problem individually. Circulate around the room providing assistance as needed and identifying students who have used different approaches for solving the problem.
20. Select students to share their solutions that represent a variety of strategies.
21. Have students select a problem scenario from among three options to complete as a homework assignment. (It may be necessary to manage the selection of scenarios to insure roughly the same number of students completing each.)

Instructional Tip:
Provide different problem scenarios to support differentiation based on student skills and needs. See Attachment H, *WASA, USA Problem*; Attachment I*, Deck Problem;* and Attachment J, *Stained Glass Window Problem* for three problem scenarios that provide a range of level of difficulty. Consider designing additional or similar problems to reflect local situations, such as a map of an area town, scale drawing of deck at teacher's home, building features of school or design of playground equipment.

Part Four:
22. Organize students by problem scenario and allow time for them to share their strategies with their small group. Create multiple groups for each scenario to keep the size of the groups manageable – two or three students per group.
23. Have groups select one of the solutions to their problem scenario to share with the class. Post each group's solutions around the room or arrange on tables/desks. Allot time for students to circulate around the room to view the various solution strategies for all three scenarios. Direct groups to identify one member to stand by the group's solution to present solution and answer questions as students circulate among the solutions. The remaining members of the group view the solutions posted by the other groups. Provide direction to students, as needed, and insure the students have time to present their solution and answer questions. Allot approximately two to three minutes per station and allow students to visit about five stations so they see a solution for each problem scenario.
24. Instruct students to look for a solution that uses a different strategy for solving the same problem they did and a solution that uses a similar strategy for solving a different problem.
25. Discuss as a class the similarities and differences among the scenarios and solution strategies.
26. Distribute Attachment C, *Post-Assessment* or a similar set of items for students to complete individually.
27. Collect student work and evaluate responses to determine student understanding and skill.

<u>Differentiated Instructional Support</u>:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Create a word wall to assist students in building vocabulary related to angle relationships.

- Provide or have student create drawings to represent more complex problem situations involving angle relationships and multiple sets of intersecting, parallel and perpendicular lines.

Extension:

- Have students design posters or bulletin boards meeting specific criteria that draw upon knowledge of and use of angle relationships within the design.
- What careers or professions use angle relationships in their work?

Home Connections:

- Have students collect pictures or create drawings that illustrate examples of situations found in the home, community or workplace, where angle relationships are used to find measures or in the design of a product or structure.
- Ask parent/guardian how they use angle relationships; e.g., why a plumb line is used when hanging wallpaper or tiling a floor?

Materials and Resources:

The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site's main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher: Overhead projector, transparencies, chart paper, markers

For the student: Protractor, tape, electrical wire ties, spaghetti, pipe cleaners or straws

Vocabulary:

- alternate exterior angles
- alternate interior angles
- corresponding angles
- intersecting lines
- parallel lines
- perpendicular lines
- right angles
- same side interior angles
- same side exterior angles
- supplementary angles
- vertical angles

I Can Name that Angle in One Measure! – Grade Eight

Technology Connections:

Use geometry software to create intersecting, perpendicular and parallel lines cut by a transversal and to verify conjectures about angles measures.

Attachments:

Attachment A, *Pre-Assessment Activity*
Attachment B, *Pre-Assessment Scoring Guidelines*
Attachment C, *Post-Assessment*
Attachment D, *Post-Assessment Answer Key and Sample Scoring Guidelines*
Attachment E, *Conjecture Form*
Attachment F, *Graphic Organizer*
Attachment G, *Railing Between Floors and Answer Key*
Attachment H, *WASA, USA and Answer Key*
Attachment I, *Deck Problem and Answer Key*
Attachment J, *Stained Glass Window Problem and Answer Key*

I Can Name that Angle in One Measure! – Grade Eight

Attachment A
Pre-Assessment Activity

Name _____ Date _____

Figure A

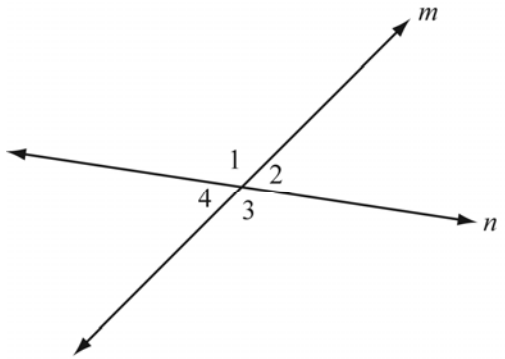


Figure B

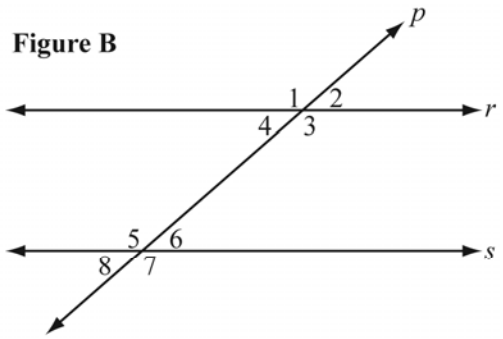


Figure C

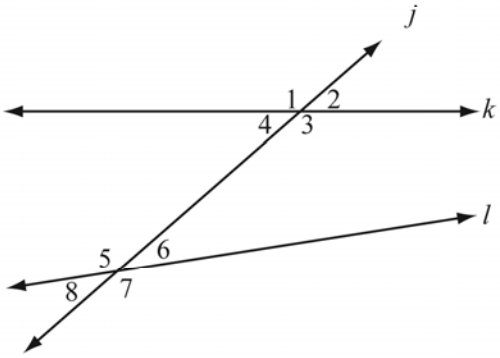


Figure D

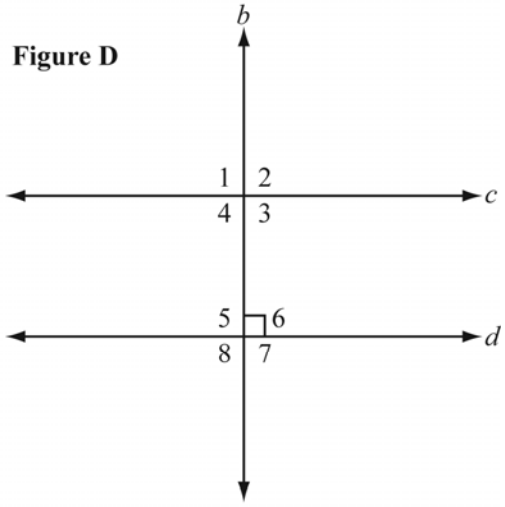
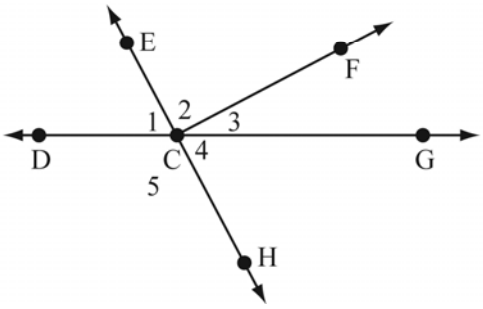


Figure E



I Can Name that Angle in One Measure! – Grade Eight

Attachment B
Pre-Assessment Scoring Guidelines

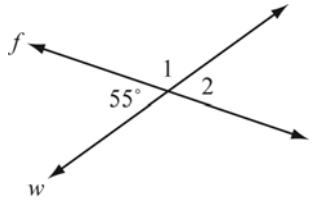
Level 3	Demonstrates concepts and skills in lesson prior to instruction: • Uses precise mathematical language to describe and classify angles, lines, and angle relationships • Generalizes angles relationships in intersecting lines, perpendicular lines, and parallel lines cut by a transversal and applies relationships to solve problems • Makes conjectures and uses mathematical reasoning to support or refute conjectures of self and others
Level 2	Demonstrates sufficient understanding and skill to begin lesson: • Uses informal language and basic mathematics terms from pervious grade to describe angles and lines • Recognizes some relationship between angles and lines • Makes conjectures; however, reasoning may be inconsistent or incomplete.
Level 1	Intervention needed prior to beginning lesson to address gaps in conceptual understanding and pre-requisite skills • Has difficulty recalling and using basic terms from previous grades to describe angles and lines. • Demonstrates weak skills in measuring angles and does not recognize or connect angles measures to the characteristics of the lines forming the angles. • Makes conjectures marginally connected or unconnected to angle and line relationships; unable to support or refute conjectures and uses frequently incomplete or highly flawed reasoning.

I Can Name that Angle in One Measure! – Grade Eight

Attachment C
Post-Assessment

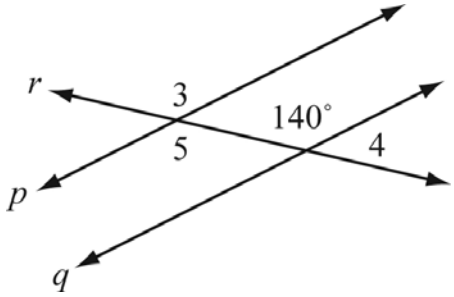
Name _____ Date _____

Part One: Use the information provided to find the measure of the angles:



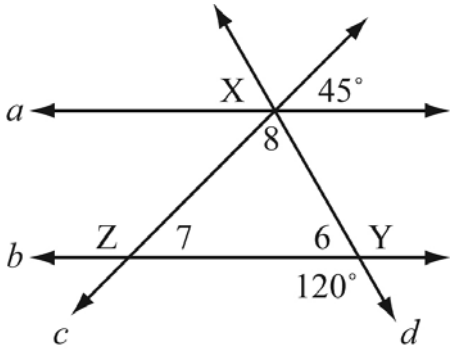
1. What is the measure of angle 1?
2. What is the measure of angle 2?

Lines *p* and *q* are parallel.



3. What is the measure of angle 3?
4. What is the measure of angle 4?
5. What is the sum of the measures of angles 3 and 5?

Lines *a* and *b* are parallel.
Lines *a*, *b* and *c* intersect at point X.
Lines *b* and *d* interest at point Y.
Lines *b* and *c* intersect at point Z.



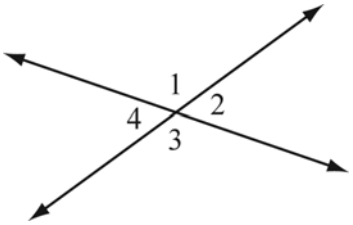
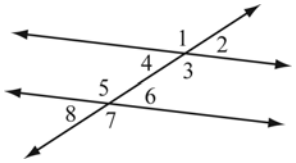
6. What is the measure of angle 6?
7. What is the measure of angle 7?
8. What is the measure of angle 8?

I Can Name that Angle in One Measure! – Grade Eight

Attachment C (Continued)

Post-Assessment

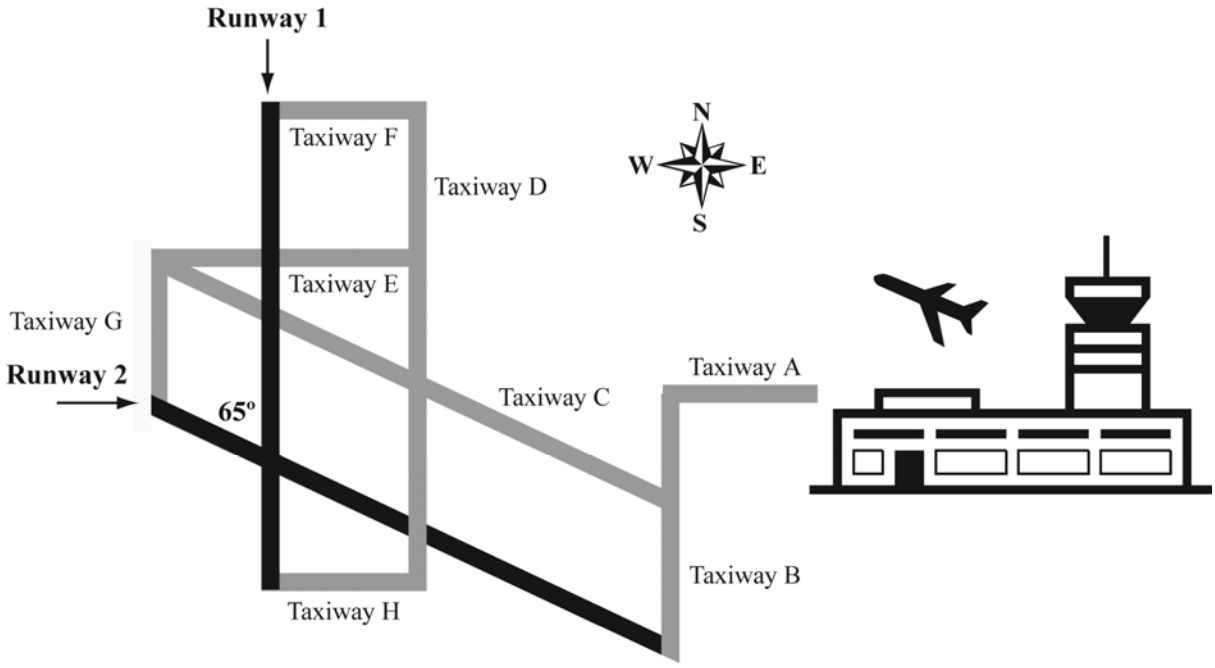
Part Two: Indicate whether each statement is ***always true***, ***sometimes true*** or ***never true***. Provide an explanation to support your answer, such as telling why you know the statement is true or drawing a sketch of a situation for which the statement would not be true.

 For ***any*** pair of intersecting lines …	**Always True**	**Sometimes True, Sometimes False**	**Never True**	**Explanation**
9. $m\angle 1 > m\angle 3$				
10. $m\angle 3 + m\angle 4 = 180^\circ$				
11. $m\angle 4 + m\angle 2 < 180^\circ$				
 For ***any*** pair of parallel lines cut by a transversal …	**Always True**	**Sometimes True, Sometimes False**	**Never True**	**Explanation**
12. $\angle 1 \cong \angle 7$				
13. $m\angle 1 + m\angle 6 > 180^\circ$				
14. $m\angle 3 + m\angle 6 = 180^\circ$				

I Can Name that Angle in One Measure! – Grade Eight

Attachment C (Continued)
Post-Assessment

Part Three: Use the diagram of the airport runways and taxiways to complete the task.



- Runway 1 is parallel to taxiways B, D and G.
- Runway 2 is parallel to taxiway C.
- Taxiway A is perpendicular to taxiway B.
- Taxiways F, E and H are perpendicular to taxiway D.
- The measure of one of the angles at the intersection of runways 1 and 2 is 65°.

An airplane is leaving the terminal. The air traffic controller needs to give directions to the pilot by giving the name of the taxiway and a description of the turn. The plane can only travel on taxiways.

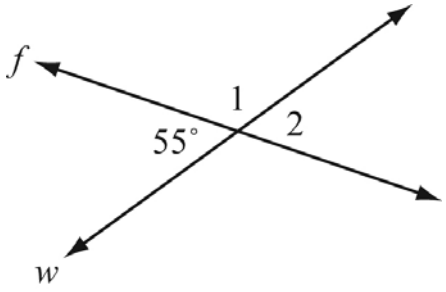
Add the steps needed to direct the plane to the north end of Runway 1 ready to travel south.

- Go west on taxiway A.
- Make a 90° turn south (or left) onto taxiway B

I Can Name that Angle in One Measure! – Grade Eight

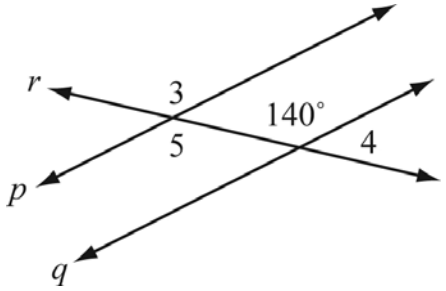
Attachment D
Post-Assessment Answer Key and Sample Scoring Guidelines

Scoring Guidelines for Part One: One point for each correct answer.



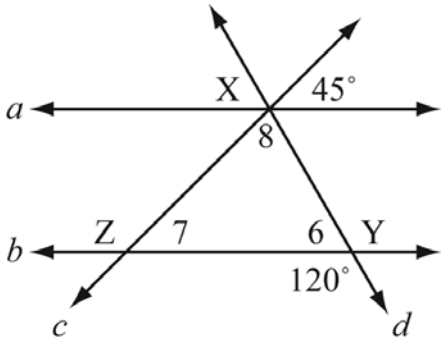
1. What is the measure of angle 1? ***125***°
2. What is the measure of angle 2? ***55***°

Lines *p* and *q* are parallel.



3. What is the measure of angle 3? ***140***°
4. What is the measure of angle 4? ***40***°
5. What is the sum of the measures of angles 3 and 5? ***280***°

Lines *a* and *b* are parallel.
Lines *a*, *b* and *c* intersect at point X.
Lines *b* and *d* interest at point Y.
Lines *b* and *c* intersect at point Z.



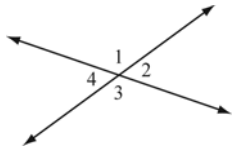
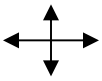
6. What is the measure of angle 6? ***60***°
7. What is the measure of angle 7? ***45***°
8. What is the measure of angle 8? ***75***°

I Can Name that Angle in One Measure! – Grade Eight

Attachment D (Continued)
Post-Assessment Answer Key and Sample Scoring Guidelines

Scoring Guidelines for Part Two: Use the following rubric to score each question.

2 points	Appropriate category indicated and accurate explanation is provided. Explanation may be communicated using words, symbols or sketches.
1 point	Demonstrates partial understanding; e.g., an appropriate category indicated, and the explanation contains accurate information related to the situation but is incomplete or contains a flaw in reasoning.
0 point	An inappropriate category indicated with a flawed explanation or no explanation. An alternate 0-point response is an appropriate category indicated with no explanation or an explanation with no correct information related to the situation and/or multiple flaws in reasoning.

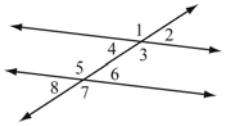
 For ***any*** pair of intersecting lines …	**Always True**	**Sometimes True, Sometimes False**	**Never True**	**Explanation** ***Explanation may vary.***
9. $m\angle 1 > m\angle 3$			**X**	**Angles 1 and 3 are vertical angles and vertical angles are always congruent.**
10. $m\angle 3 + m\angle 4 = 180^\circ$	**X**			**Adjacent angles formed by intersecting lines are supplementary.**
11. $m\angle 4 + m\angle 2 < 180^\circ$		**X**		**This is true when both angles are right angles, like this** 

I Can Name that Angle in One Measure! – Grade Eight

Attachment D (Continued)
Post-Assessment Answer Key and Sample Scoring Guidelines

Scoring Guidelines for Part Two: Use the following rubric to score each question.

2 points	Appropriate category indicated and accurate explanation is provided. Explanation may be communicated using words, symbols or sketches.
1 point	Demonstrates partial understanding; e.g., an appropriate category indicated, and the explanation contains accurate information related to the situation but is incomplete or contains a flaw in reasoning.
0 point	An inappropriate category indicated with a flawed explanation or no explanation. An alternate 0-point response is an appropriate category indicated with no explanation or an explanation with no correct information related to the situation and/or multiple flaws in reasoning.

 For ***any*** pair of parallel lines cut by a transversal …	**Always True**	**Sometimes True, Sometimes False**	**Never True**	**Explanation** ***Explanation may vary.***
12. $\angle 1 \cong \angle 7$	**X**			$\angle 7 \cong \angle 5$ because vertical angles are congruent. $\angle 5 \cong \angle 3$ because alternate interior angles are congruent. $\angle 7 \cong \angle 3$ because they are both congruent to the same angle. $\angle 3 \cong \angle 1$ because they are vertical angles. So, $\angle 1 \cong \angle 7$ because they are congruent to the same angle.
13. $m\angle 1 + m\angle 6 > 180^\circ$			**X**	$\angle 6 \cong \angle 2$ and $\angle 2$ and $\angle 1$ are supplementary. Therefore, the sum of the measures of $\angle 6$ and $\angle 1$ must be 180°
14. $m\angle 3 + m\angle 6 = 180^\circ$	**X**			$m\angle 3 = m\angle 5$ because they are alternate interior angles and have the same measures. $\angle 5$ and $\angle 6$ form a straight line and add up to 180°, so $\angle 3$ and $\angle 6$ also add up to 180°.

I Can Name that Angle in One Measure! – Grade Eight

Attachment D (Continued)
Post-Assessment Answer Key and Sample Scoring Guidelines

Scoring Guidelines for Part Three:

Sample correct responses:
Add the steps needed to direct the plane to the north end of Runway 1:

- Go west on taxiway A.
- Make a 90° turn south (or left) onto taxiway B

Sample Response A:	***Sample Response B:***
▫ ***Make a 115° turn northwest (or right) onto taxiway C***	▫ ***Make a 115° turn northwest (or right) onto taxiway C***
▫ ***Make a 65° turn north (or right) onto taxiway D***	▫ ***Make a 155° turn east (or right) onto taxiway E***
▫ ***Make a 90° turn west (or left) onto taxiway F***	▫ ***Make a 90° turn north (or left) onto taxiway D***
▫ ***Make a 90° turn south (or left) onto runway 1***	▫ ***Make a 90° turn west (or left) onto taxiway F***
	▫ ***Make a 90° turn south (or left) onto runway 1***

3 points	Response contains complete and accurate directions, including specifying all taxiways and clearly stating the number of degrees and direction for all turns
2 points	Response contains directions that have one gap or flaw in labeling or reasoning. For example, the response may ▫ identify all taxiways and degrees of turns, but does not give direction of turns ▫ include one incorrect turn or direction; however, remaining directions will get plane to correct position based on the error ▫ send the plane to wrong end of runway 1; however, all directions are accurate and clearly stated.
1 point	Response contains partial directions or major gaps/flaws. For example the response may: ▫ identify the taxiway and direction of turns accurately, but fails to identify number of degrees for all turns or makes multiple errors in identify the number of degrees ▫ include at least two correct directions/turns with no more than two incorrect or missing directions/turns
0 point	Response is incomplete or highly flawed. For example, the response may: ▫ identify only one direction/turn accurately ▫ lack sufficient detail; e.g., may give taxiway letters only with no attempt to describe the turns

I Can Name that Angle in One Measure! – Grade Eight

Attachment E
Conjecture Form

Name _____ Date _____

"Check" the appropriate column to indicate if the conjecture is always true, sometimes true or never true. Provide a sketch or describe a situation to support your answer.

Conjecture	Always True	Sometimes True	Never True	Supporting Evidence
For intersecting lines:				
For any two lines cut by a transversal:.				

I Can Name that Angle in One Measure! – Grade Eight

Attachment E(Continued)
Conjecture Form
Sample Conjectures

"Check" the appropriate column to indicate if the conjecture is always true, sometimes true or never true. Provide a sketch or describe a situation to support your answer.

	Always True	Sometimes True	Never True	Supporting Evidence
For intersecting lines: The measures of any two adjacent angles are supplementary.				
For any two lines cut by a transversal: If I know the measure of one angle, I can find the measures of the remaining seven angles without using a protractor.				
If I know two corresponding angles are congruent then I know the lines must be parallel.				
The measures of alternate exterior angles are greater than the measures of alternate interior angles.				
The sum of the measures of alternate interior angles is always 180 degrees.				
Vertical angles are not congruent.				

I Can Name that Angle in One Measure! – Grade Eight

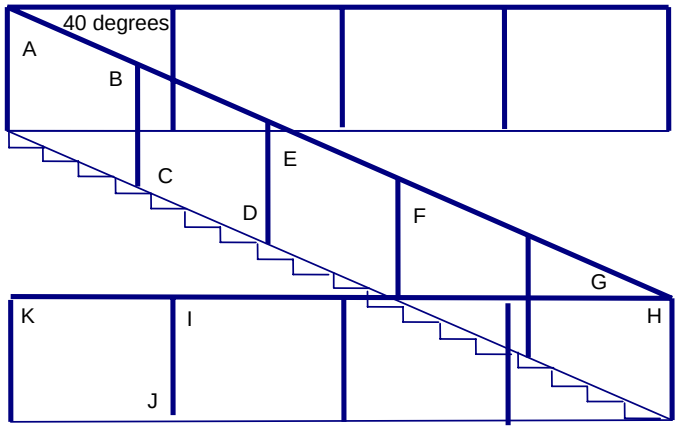
Attachment F
Graphic Organizer

Name _____ Date _____

	Angle Relationships We Know to be True	Sketch	Real-World Example
Intersecting Lines	Description Symbolic Representation		
Perpendicular Lines	Description Symbolic Representation		
Parallel Lines with Transversal	Description Symbolic Representation		
	Description Symbolic Representation		
	Description Symbolic Representation		

I Can Name that Angle in One Measure! – Grade Eight

Attachment G
Railing Between Floors



Hint: You may want to extend some of the lines in order to recognize the specific types of angles

You are ordering a railing (the highlighted segments) for the staircase illustrated to the left. This railing will be constructed off-site and brought in to be installed in one piece.

The vertical sections are perpendicular to the floors.

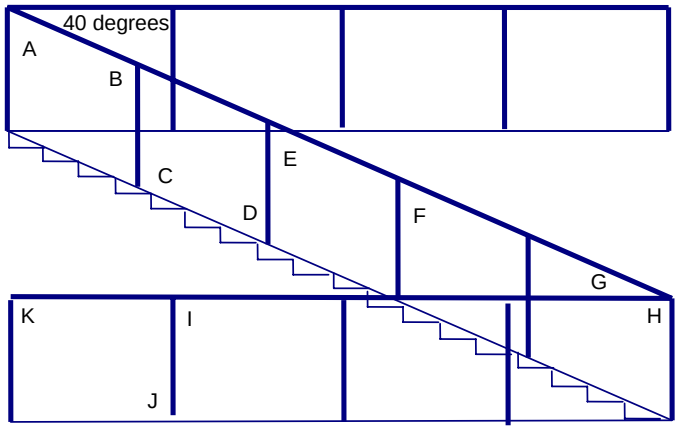
The longest top railing that runs between the two floors makes a 40 degree angle with the top railing as indicated in the diagram.

Find the measures of all the labeled angles and justify your measurements by identifying the type of angle created. (ie. alternate interior angles are congruent when parallel lines are cut by a transversal)

Angle	Measure of angle	Reasoning to support your answer
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		

I Can Name that Angle in One Measure! – Grade Eight

Attachment G (Continued)
Railing Between Floors Answer Key



Hint: You may want to extend some of the lines in order to recognize the specific types of angles

You are ordering a railing (the highlighted segments) for the staircase illustrated to the left. This railing will be constructed off-site and brought in to be installed in one piece.

The vertical sections are perpendicular to the floors.

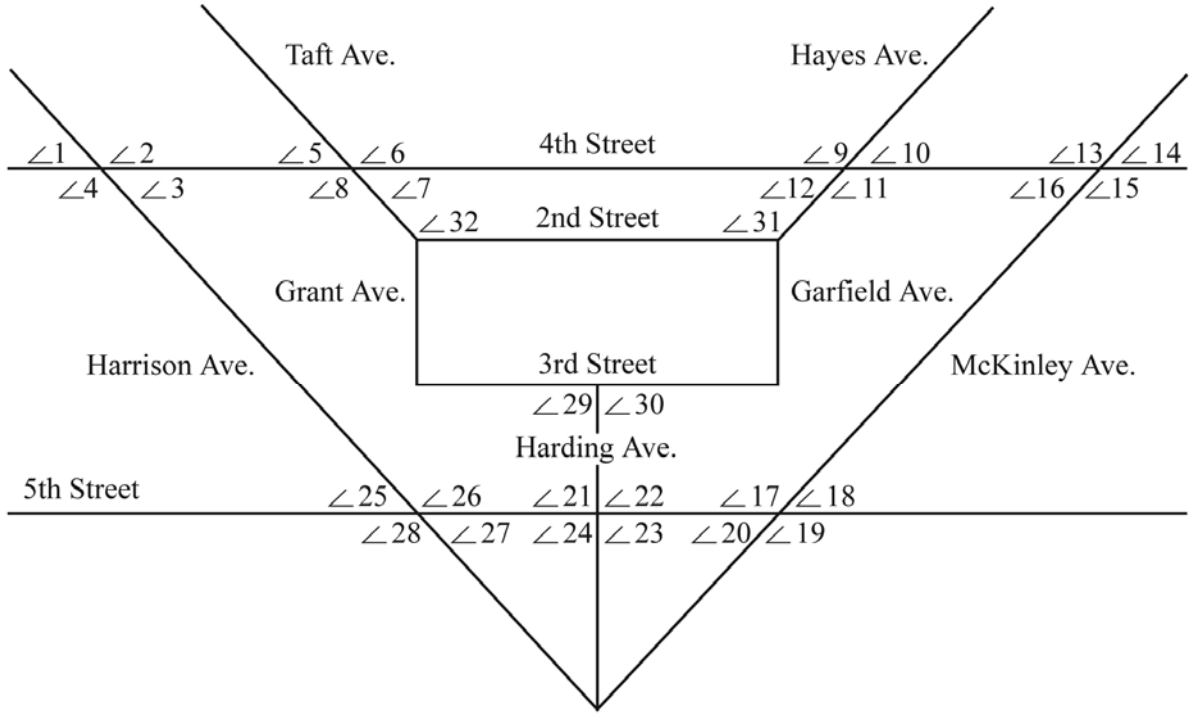
The longest top railing that runs between the two floors makes a 40 degree angle with the top railing as indicated in the diagram.

Find the measures of all the labeled angles and justify your measurements by identifying the type of angle created. (ie. alternate interior angles are congruent when parallel lines are cut by a transversal)

Angle	Measure	Reasoning to support your answer
A	50	Complementary angles (90-40=50)
B	130	Same side interior ∠'s are supplementary with // lines
C	130	Alternate interior ∠'s are congruent with // lines
D	50	Same side interior ∠'s are congruent with // lines
E	50	Corresponding ∠'s congruent or alt. Int. ∠'s congruent with // lines
F	50	Same side exterior ∠'s supplementary or corresponding ∠'s congruent with // lines
G	40	Alternate interior ∠'s congruent with // lines
H	90	// lines cut by a perpendicular line form right angles
I	90	Same as above or same side interior are supplementary with // lines
J	90	Same as H or alternate interior ∠'s are congruent with parallel lines
K	90	Same as H or corresponding ∠'s are congruent with parallel lines

I Can Name that Angle in One Measure! – Grade Eight

Attachment H
WASA, USA



m∠1 = 84 m∠12 = 48 m∠22 = 90

Numbered streets are parallel (2nd, 3rd, 4th, 5th).
Harrison Ave. is parallel to Taft Ave.
Hayes Ave. is parallel to McKinley Ave.

Angle	Measurement	Why?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

I Can Name that Angle in One Measure! – Grade Eight

Attachment H (Continued)
WASA, USA

14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		
26		
27		
28		
29		
30		
31		
32		

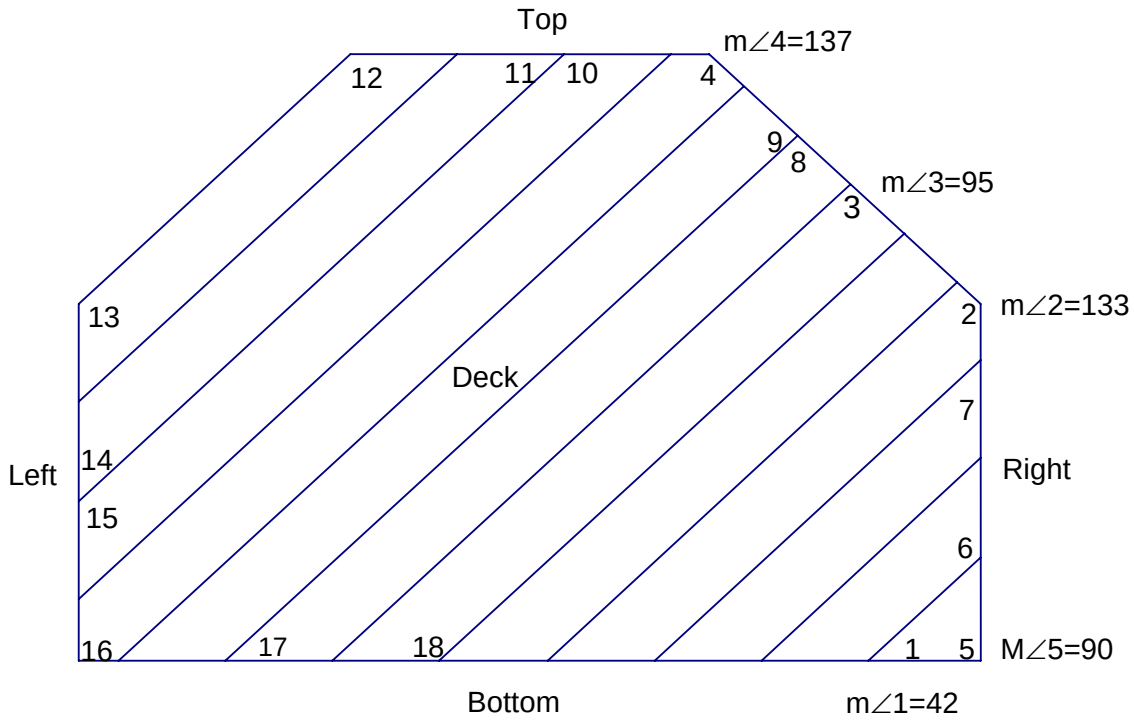
I Can Name that Angle in One Measure! – Grade Eight

Attachment H(Continued)
WASA, USA Answer Key

Angle	Measurement	Why?
1	*84*	*Given*
2	*96*	*Supplementary to angle 1*
3	*84*	*Vertical with angle 1*
4	*96*	*Vertical with angle 2*
5	*84*	*Corresponds with angle 1*
6	*96*	*Corresponds with angle 2*
7	*84*	*Alternate exterior with angle 1*
8	*96*	*Alternate interior with angle 2*
9	*132*	*Supplementary with angle 10*
10	*48*	*Vertical with angle 12*
11	*132*	*Vertical with angle 9*
12	*48*	*Given*
13	*132*	*Corresponds with angle 9*
14	*48*	*Alternate exterior with angle 12*
15	*132*	*Corresponds with angle 11*
16	*48*	*Alternate interior with angle 10*
17	*132*	*Corresponds with angle 13*
18	*48*	*Alternate interior with angle 16*
19	*132*	*Alternate exterior with angle 13*
20	*48*	*Corresponds with angle 16*
21	*90*	*Supplementary with angle 22*
22	*90*	*Given*
23	*90*	*Vertical with angle 21*
24	*90*	*Vertical with angle22*
25	*84*	*Corresponds with angle 1*
26	*96*	*Alternate interior with angle 4*
27	*84*	*Vertical with angle 25*
28	*96*	*Vertical with angle 26*
29	*90*	*Alternate interior with angle 22*
30	*90*	*Alternate interior with angle 21*
31	*132*	*Supplementary with angle 12 or corresponds to angle 9*
32	*96*	*Supplementary with angle 7 or corresponds to angle 6*

I Can Name that Angle in One Measure! – Grade Eight

Attachment I
Deck Problem



GIVEN:

- The top side and the bottom side are parallel.
- The left side and the right side are parallel.
- m∠1=42
- m∠2=133
- m∠3=95
- m∠4=137
- m∠5=90

Find:

m∠6 =_____ m∠10=_____ m∠15=_____

m∠7 =_____ m∠11=_____ m∠16=_____

m∠8 =_____ m∠12=_____ m∠17=_____

m∠9 =_____ m∠13=_____ m∠18=_____

m∠14=_____

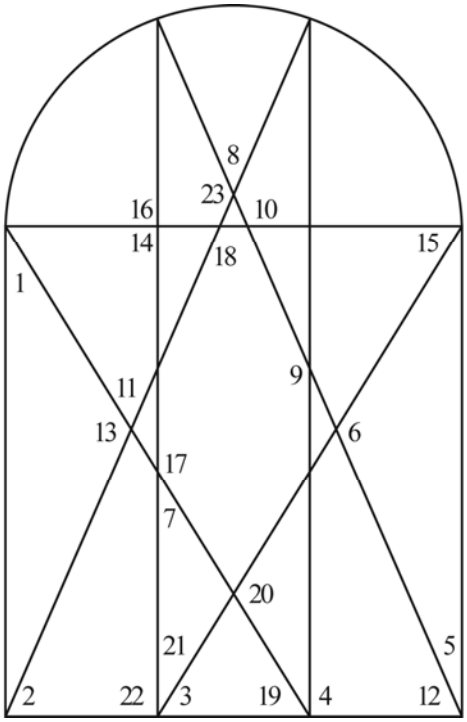
I Can Name that Angle in One Measure! – Grade Eight

Attachment I (Continued)
Deck Problem Answer Key

Angle	Measure of the angle	Reasoning to support your answer
6	*132*	The sum of angles in a triangle is 180°. 180 – (90 + 42) = 48. The angle adjacent to angle 6 is 48 degrees.
7	*48*	Same side interior supplementary with // lines
8	*95*	Corresponding ∠with ∠3 are congruent with // lines
9	*85*	Alternate Ext. with ∠ 3 are supplementary with // lines
10	*138*	Same side interior supplementary (need to extend board and bottom side of deck)
11	*42*	Linear pair with ∠ 10 are supplementary
12	*138*	Corresponding with ∠ 10 are congruent with // lines
13	*132*	Corresponding with ∠ 15 are congruent with // lines
14	*48*	Same side interior with ∠ 13 are supplementary with // lines
15	*132*	Linear pair with the 3rd ∠ of triangle created by extending the board and the bottom side of the deck
16	*90*	Same side interior with ∠ 5 supplementary with // lines
17	*42*	Corresponding with ∠ 1 are congruent with // lines
18	*138*	Same side interior with ∠ 17 supplementary with // lines

I Can Name that Angle in One Measure! – Grade Eight

Attachment J
Stained Glass Window Problem



$m\angle 1 = 35$

$m\angle 2 = 65$

$m\angle 3 = 55$

$m\angle 4 = 90$

$m\angle 12 = m\angle 2$

Vertical lines are parallel.

Horizontal lines are parallel.

Vertical lines are perpendicular to horizontal lines.

Angle	Measure	Reasoning to support your answer (why?)
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		

I Can Name that Angle in One Measure! – Grade Eight

Attachment J (Continued)

Stained Glass Window Problem Answer Key

Angle	Measure	Reasoning to support your answer (why?)
5	25	Students may use a variety of strategies to calculate and determine measures. Accept appropriate reasoning.
6	120	
7	35	
8	50	
9	155	
10	115	
11	60	
12	65	
13	120	
14	90	
15	55	
16	90	
17	145	
18	125	
19	55	
20	110	
21	35	
22	90	
23	130	

Note: This is the most difficult of the three worksheets. Students need to solve many of the unnumbered angles in triangles to solve for missing angles.