Solving and Graphing Inequalities
Up and Away

ACTIVITY 2.5

SUGGESTED LEARNING STRATEGIES: Marking the Text, Shared Reading, Create Representations, Think/Pair/Share

Geri wants to become a commercial airline pilot someday. She found this information while doing research on this career.

- The airplane's captain must be at least 23 years old.
- The captain must have a minimum of 1,500 hours of flying experience.
- By law pilots can fly a maximum of 100 hours in a month.
- By law pilots may not fly more than 32 hours during any 7 consecutive days.

Each piece of information that Geri found can be modeled best by using an **inequality**. Phrases like *at least, no less than,* and *a minimum of* are used to indicate that one quantity is greater than or equal to another. Phrases like *at most*, *no more than,* and *a maximum of* are used to indicate that one quantity is less than or equal to another.

Equations and inequalities that contain variables are **open sentences**. Use these open sentences to answer Item 1.

$x > 20$ $x < 20$ $x \leq 20$ $x \geq 20$

1. For each situation below, write the open sentence that represents it. Some of the open sentences will be used more than once. Then tell what x represents—this is called defining the variable.

a. She finished the license test in no more than 20 minutes.
_____, x represents _____

b. The temperature was less than 20°F the day of the test.
_____, x represents _____

c. More than 20 students were in her flight school class.
_____, x represents _____

d. No one under 20 is admitted to the flight school.
_____, x represents _____

e. No more than 20 students will get a job with the airline.
_____, x represents _____

f. Training uniforms cost at least $20.
_____, x represents _____

My Notes

ACADEMIC VOCABULARY

An **inequality** is a mathematical statement showing that one quantity is greater than or less than another.
Inequalities use these symbols:

$>$ is greater than

$<$ is less than

$\geq$ is greater than or equal to

$\leq$ is less than or equal to

ACADEMIC VOCABULARY

An **open sentence** is an equation or inequality with variables. An open sentence can be true or false, depending on what values are substituted for the variables.

SUGGESTED LEARNING STRATEGIES: Create Representations, Vocabulary Organizer

My Notes

MATH TIP

In a correctly stated inequality the symbol points toward the smaller number.

$2 < 5$; 2 is smaller

$2 > 0$; 0 is smaller

2. Write an inequality to describe each situation. Define your variable by explaining what it represents.

a. An airplane's captain must be at least 23 years old.

b. An airplane's captain must have a minimum of 1,500 hours of flying experience.

c. A pilot may not fly more than 32 hours during any 7 consecutive days.

d. A pilot can fly a maximum of 100 hours in a month.

e. Geri's age is less than 23.

A **solution of an inequality** is any number that produces a true statement when it is substituted for the variable in the inequality. One possible solution to the equation $p < 25$ is 18 because $18 < 25$ is a true statement.

3. Commercial airplanes are required to fly at least 1500 feet above the highest fixed object in a residential area. The highest building in Geri's town is 240 feet.

a. Write an inequality to describe this situation. Define your variable.

b. Find three possible solutions to the inequality you wrote in part a.

c. List one value that is not a solution.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Create Representations

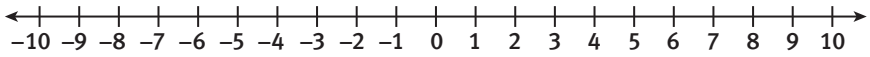
My Notes

Consider the inequalities $x > 2$ and $x < 8$. These two inequalities can be combined like this: $2 < x < 8$. Combining inequalities in this way forms a **compound inequality**.

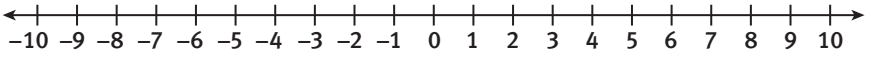
4. Use words to describe $2 < x < 8$. Then list six values of x that satisfy the inequality: $2 < x < 8$.

Inequalities that are open sentences usually have many solutions. The solutions are often easier to graph than to list. Inequalities with one variable are graphed on a number line.

5. Graph all solutions to $a > 3$.

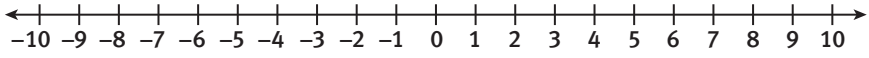


6. Graph all solutions to $y \leq -1$



7. Consider the compound inequality $-3 \leq w < 2$. This inequality can be expressed as $w \geq -3$ and $w < 2$.

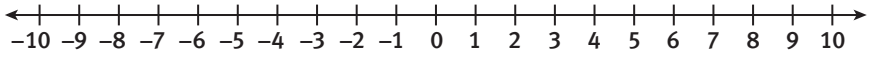
a. Graph $w \geq -3$ on the number line below.



b. Graph $w < 2$ on the same number line but in a different color.

c. What do you notice about the intersection of these two graphs?

d. Graph the solution to $-3 \leq w < 2$ on the number line below.



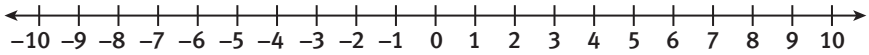
e. Describe the graph of the solution.

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations

8. Consider $w < -3$ or $w \geq 2$.

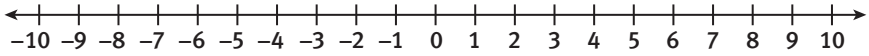
a. Graph $w < -3$ on the number line below.



b. Graph $w \geq 2$ in a different color on the same number line.

c. When you graph a compound inequality with the word *or*, what will the graph be?

d. Graph the solution of $w < -3$ or $w \geq 2$ on the number line below.



MATH TIP

One way of solving equations is to use inverse operations.

EXAMPLE 1

As inequalities become more complex, you need a systematic way to solve them just as with equations. Use the rules for solving equations to solve the inequality $w + 3 \leq 8$. Check your answer.

Step 1: *Write the inequality.*

$w + 3 \leq 8$

Step 2: *Subtract 3 from both sides.*

$w + 3 - 3 \leq 8 - 3$

$w \leq 5$

Solution: $w \leq 5$, so any number ≤ 5 is a solution.

Check: Choose a value for w that satisfies the solution.

Use 2.

Substitute the chosen value for w in the original inequality.

$2 + 3 \leq 8$

$5 \leq 8$ ✔

TRY THESE A

Write your answers in the My Notes space. Show your work.

Solve each inequality.

a. $z - 7 \leq -2$ **b.** $3m > 15$ **c.** $\frac{y}{5} < 4$

SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

In some cases the rules for solving equations will not work for solving inequalities.

EXAMPLE 2

Solve the inequality $-2a \leq 6$.

Step 1: *Write the inequality.*

$$-2a \leq 6$$

Step 2: *Divide both sides by –2.*

$$\frac{-2a}{-2} \leq \frac{6}{-2}$$

Solution: $a \leq -3$

Check: Choose a value for a that satisfies the solution. Use -5.
Substitute the chosen value for a in the original inequality.

$$-2(-5) \leq 6$$

$$10 \leq 6$$

Since the chosen value does not make the inequality true, the solution is not correct.

The rules for solving an equation do not work for solving an inequality when you must multiply or divide by a negative number. In these situations, you must reverse the inequality symbol in the solution. In the example above, the solution should be $a \geq -3$. Choosing a value that satisfies this solution will make the original inequality true.

Check: Choose a value for a that satisfies $a \geq -3$. Use 5.

Substitute 5 for a in the original inequality.

$$-2(5) \leq 6$$

$$-10 \leq 6$$

TRY THESE B

Write your answers in the My Notes space. Show your work.

Solve. $\frac{x}{-3} > 4$ $\qquad -4x \geq -12$

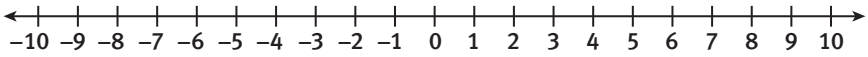
My Notes

SUGGESTED LEARNING STRATEGIES: Quickwrite, Create Representations, Shared Reading, Summarize/Paraphrase/Retell

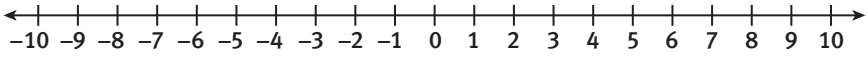
9. Do the rules for solving equations work for solving inequalities? Explain why or why not.

10. Solve each inequality. Graph the solution.

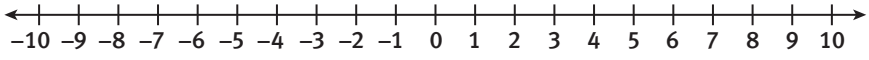
a. $\frac{1}{3}x + 2 \geq -1$



b. $-5x - 3 \leq 7$



c. $-3x > 15$



11. The pilot of a small plane wanted to stay at least 1600 feet beneath some storm clouds at 11,200 feet. After takeoff, for how long could the pilot ascend at a rate of 640 ft/min?

MATH TIP

Question 11 can be modeled with the inequality $640t + 1600 \leq 11{,}200$ where t is the number of minutes that the pilot ascends at 640 ft/min. Solve $640t + 1600 \leq 11{,}200$. Be sure to check your solution.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write an inequality to describe each situation. Define your variable.

a. You must be at least 4.4 feet tall to ride the kiddie roller coaster.

b. You must drive between 45 mph and 65 mph inclusive on the highway.

c. You may have no more than one C on your report card to be eligible to play sports.

2. Graph all solutions to each inequality.

a. $x > 6$ **b.** $y \leq -4$

c. $2 < w < 7$ **d.** $x < 4$ or $x \geq 6$

3. Solve each inequality

a. $x + 5 \geq 11$ **b.** $y - 2.2 < 1.7$

c. $-8 + 3x > 1$ **d.** $-2x - 1 \geq 5$

4. Karen has a $150 e-gift card. She wants to buy sweaters. Each sweater costs $27 and there is a $10.95 shipping charge per order. Write and solve an inequality to find the number of sweaters that Karen can purchase. Define your variable.

5. MATHEMATICAL REFLECTION Think about something in your own life that might be represented as an inequality. Describe the situation and write an inequality for that situation.